

# *P-Q* Circle Diagram Based Parameter Measurement for Permanent Magnet Synchronous Motor Including Iron Loss

## Abstract

This paper presents parameter measurement for permanent magnet synchronous motors based on the *P-Q* circle diagram. Three electrical parameters of permanent magnet synchronous motors, i.e., the equivalent iron loss resistance, armature inductance, and electrical motive force (emf) coefficient are simultaneously measured. The advantages of this method are that it can be implemented under constant excitation and it dispenses with the generating test for the emf coefficient. The proposed method is applied to a 160W permanent magnet synchronous motor, and then the measurement results are analyzed.

**Key words:** Iron loss, Parameter measurement, Permanent magnet synchronous motor, *P-Q* circle diagram

## I. INTRODUCTION

Traditionally, vector controlled ac drives have been performed under the assumption that there is no iron loss in motors. As the employment of vector controlled ac motors, especially induction motor, permanent magnet synchronous motor (PMSM), synchronous reluctance motor has become standard in industrial drives, the improvement of ac motor drives has been important issue. For this reason, several authors have made an attempt to consider the iron loss in vector controlled ac motor drives. The vector controls including the iron loss are investigated for induction motors [1]-[3] and synchronous reluctance motors [4], [5]. The authors have derived a mathematical model of PMSM considering the iron loss under the assumption that the iron loss is produced in equivalent circuit for eddy current. The derived voltage equation newly possesses curious parameter referred to as the equivalent iron loss resistance and the parameter is connected in series with the armature resistance [6]. However, the behavior of the equivalent iron loss resistance has not been revealed experimentally. The objective of this paper is to measure the electrical parameters for this mathematical model and investigate the parameter characteristics with respect to driving frequency in steady-state condition.

As compared with induction motors, parameter measurement for PMSM is a troublesome job because field excitation is constant, i.e., permanent magnet flux is constant. Especially, the iron loss resistance cannot be measure by the

no-load test. This paper presents parameter measurement for PMSM based on the *P-Q* circle diagram [7]. The *P-Q* circle diagram can be depicted from the pairs of active power and reactive power when PMSM operates in various load conditions under the constant voltage and constant frequency. Electrical parameters of PMSM can easily be calculated from the center and radius of the *P-Q* circle diagram. Three electrical parameters of PMSM, i.e., the armature inductance, emf coefficient and equivalent iron loss resistance are simultaneously measured. The advantages of this method are that it can be implemented under constant field excitation and it dispenses with the generating test for the emf coefficient. The proposed method is applied to a 160W laboratory PMSM, and then the characteristics of driving frequency for these parameters are analyzed.

This paper is organized as follows. The mathematical model of PMSM including the iron loss is reviewed in Section 2. Parameter measurement method by using the *P-Q* circle diagram is presented in Section 3. Parameter measurement results and parameter characteristics are illustrated in Section 4. Some conclusions are given in Section 5.

## II. MATHMATHICAL MODEL OF PMSM INCLUDING IRON ROSS

Fig. 1 shows the *d-q* winding model of a PMSM. In this figure,  $1d$  and  $1q$  represent the armature *d-q* windings,  $2d$  the constant field winding corresponding to the permanent magnet mounted on the rotor and  $3d$  and  $3q$  the equivalent windings for eddy current. In this paper, the iron loss arises from the resistance of the equivalent winding for eddy current [1].

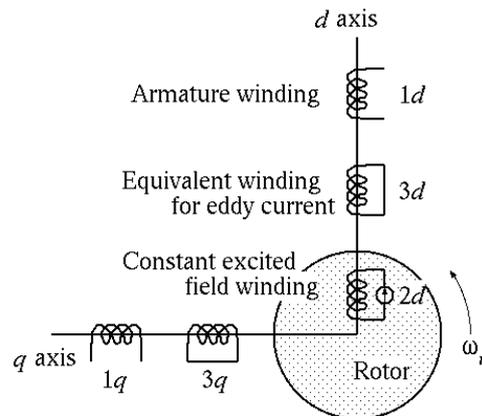


Fig. 1. *d-q* winding model of a cylindrical PMSM.

From the  $d$ - $q$  winding model, the voltage equation for each winding in steady-state condition is expressed as [6].

$$\begin{bmatrix} V_{1d} \\ V_{1q} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & -\omega_r L_1' & 0 & -\omega_r M_{13} \\ \omega_r L_1' & R_1 & \omega_r M_{13} & 0 \\ 0 & -\omega_r M_{13} & R_3 & -\omega_r L_3 \\ \omega_r L_3 & 0 & \omega_r L_3 & R_3 \end{bmatrix} \begin{bmatrix} I_{1d} \\ I_{1q} \\ I_{3d} \\ I_{3q} \end{bmatrix} + \omega_r \begin{bmatrix} 0 \\ M_{12} \\ 0 \\ M'_{32} \end{bmatrix} I_{2d} \quad (1)$$

where,  $V_1$  and  $I_1$  denote the armature voltage and current, respectively.  $I_2$  and  $I_3$  denote the equivalent field current and equivalent eddy current, respectively.  $\omega_r$  is the electrical angular velocity.  $R$  and  $L$  denote the each winding resistance and self-inductance, respectively,  $M$  is the mutual inductance between windings, and each subscript expresses each winding. Equation (1) is the form that the equivalent eddy currents remain. Since the equivalent eddy currents cannot directly be detected, the equivalent eddy currents should be eliminated by succeeding manipulations.

From (1), the equivalent eddy currents are straightforwardly derived as follows:

$$I_{3d} = \frac{(\omega_r^2 M_{13} L_3 I_{1d} - \omega_r M_{13} R_3 I_{1q} + \omega_r^2 L_3 M'_{32} I_{2d})}{(R_3^2 + \omega_r^2 L_3^2)} \quad (2)$$

$$I_{3q} = \frac{(\omega_r^2 M_{13} L_3 I_{1q} - \omega_r M_{13} R_3 I_{1d} + \omega_r R_3 M'_{32} I_{2d})}{(R_3^2 + \omega_r^2 L_3^2)} \quad (3)$$

Substituting (2) and (3) in (1), the order of the voltage equation can be reduced as,

$$\begin{bmatrix} V_{1d} \\ V_{1q} \end{bmatrix} = \begin{bmatrix} R_1 + R_m & -\omega_r L_1 \\ \omega_r L_1 & R_1 + R_m \end{bmatrix} \begin{bmatrix} I_{1d} \\ I_{1q} \end{bmatrix} + \omega_r \begin{bmatrix} M_{32d} \\ M_{12} + M_{32q} \end{bmatrix} I_{2d} \quad (4)$$

where,

$$R_m = \frac{\omega_r^2 M_{13}^2}{R_3^2 + \omega_r^2 L_3^2} R_3, \quad L_1 = L_1' - \frac{\omega_r^2 M_{13}^2}{R_3^2 + \omega_r^2 L_3^2} L_3$$

$$M_{32d} = \frac{\omega_r M_{13} R_3}{R_3^2 + \omega_r^2 L_3^2} M'_{32}, \quad M_{32q} = \frac{\omega_r^2 M_{13} L_3}{R_3^2 + \omega_r^2 L_3^2} M'_{32}$$

The parameter  $R_m$  is referred to as the equivalent iron loss resistance and is connected in series with the armature resistance  $R_1$ .  $L_1$  denotes the armature inductance. Assuming that the mutual inductances  $M_{32d}$  and  $M_{32q}$  are sufficiently small as compared with the mutual inductance  $M_{12}$ , these

inductances are neglected; as a result, the voltage equation is simplified as,

$$\begin{bmatrix} V_{1d} \\ V_{1q} \end{bmatrix} = \begin{bmatrix} R_1 + R_m & -\omega_r L_1 \\ \omega_r L_1 & R_1 + R_m \end{bmatrix} \begin{bmatrix} I_{1d} \\ I_{1q} \end{bmatrix} + \omega_r \begin{bmatrix} 0 \\ K_e \end{bmatrix} \quad (5)$$

where,  $K_e (= M_{12} I_{2d})$  represents the emf coefficient. The validity of this simplification is confirmed by the experimental result illustrated in Section 4.

### III. PARAMETER MEASUREMENT BY P-Q CIRCLE DIAGRAM

The electrical parameters of PMSM are measured based on the  $P$ - $Q$  circle diagram. In this method, external load is gradually applied to PMSM under terminal voltage with constant amplitude and constant frequency. The input active and reactive powers for each load are measured at that time. Then, the  $P$ - $Q$  circle diagram is depicted from the pairs of active and reactive powers.

Three electrical parameters including the equivalent iron loss resistance  $R_m$  are calculated from the center and radius of the  $P$ - $Q$  circle diagram. The following illustrates the relationship between the  $P$ - $Q$  circle diagram and the electrical parameters. Fig. 2 shows the vector diagram for PMSM in steady-state condition. In this figure,  $\mathbf{V}$  and  $\mathbf{I}$  denote voltage vector and current vector, respectively.

The input active power  $P$  and reactive power  $Q$  are expressed as,

$$P = VI \cos \varphi \quad (6)$$

$$Q = VI \sin \varphi \quad (7)$$

where,  $V = \sqrt{V_{1d}^2 + V_{1q}^2}$ ,  $I = \sqrt{I_{1d}^2 + I_{1q}^2}$  and  $\varphi$  is the power factor angle.

### IV. MEASUREMENT RESULTS

In this section, parameter measurement results based on the  $P$ - $Q$  circle diagram are presented and their characteristics with respect to driving frequency are also illustrated. The specifications of the PMSM utilizing in this research are listed in Table 1. Fig. 4 shows the experimental system to execute parameter measurement based on the  $P$ - $Q$  circle diagram. In order to measure the electrical parameters on various driving frequencies, a PWM inverter with 7.5 kHz carrier frequency is utilized. A digital power meter detects (DPM) the active power, reactive power, and input voltage. Load conditions for the PMSM is adjusted by changing the external resistance through the generator (PMSG) that connects directly to the PMSM.

TABLE I  
MOTOR SPECIFICATIONS

Rated power	160 W
Rated torque	0.5N·m
Rated speed	3000 rpm
Number of pole-pairs	2

#### A. Equivalent iron loss resistance

The parameter measurement results for the equivalent iron loss resistance  $R_m$ , which is introduced newly in voltage equation of PMSM, is discussed in this subsection. The measurement is executed in wide frequency range from 40Hz to 100Hz under various  $V/f$  ratios. As discussed in Section 3, the equivalent iron loss resistance is calculated by using the center of the  $P$ - $Q$  circle diagram depicted on each driving frequency.

Fig. 6 shows the equivalent iron loss resistance versus driving frequency. As can be seen from this figure, the equivalent iron loss resistance increases with increasing driving frequency. In order to explain this reason, two assumptions are introduced:

#### B. Armature inductance

The parameter measurement for the armature inductance  $L_1$  is also executed in wide frequency range from 40Hz to 100Hz under various  $V/f$  ratios. The armature inductance is calculated by using the center of the  $P$ - $Q$  circle diagram depicted on each driving frequency.

### V. CONCLUSIONS

This paper proposes parameter measurement for PMSM including iron loss. In the proposed method, three electrical parameters of PMSM, i.e., the equivalent iron loss resistance, armature resistance and emf coefficient are simultaneously measured based on  $P$ - $Q$  circle diagram. This method can be applied to motor with constant field excitation machines such as PMSM. In addition, it dispenses with the generating test for the emf coefficient. The proposed method is applied to a 160W permanent magnet synchronous motor, and then the measurement results are analyzed. The equivalent iron loss resistance increases with increasing driving frequency while the armature inductance and emf coefficient are almost constant. The validity of this proposed method is confirmed by the comparison with the traditional method.

### APPENDIX

Motor data:  
 $R_a = 2.8 \Omega$ ,

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